

Simplifying indices

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
----------------------------	--

Example 2 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	<ol style="list-style-type: none"> 1 Use the rule $a^m \times a^n = a^{m+n}$ 2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$
--	---

Example 3 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged
------------------------------------	--

Example 4 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\begin{aligned} \frac{4}{\sqrt{x}} &= \frac{4}{x^{\frac{1}{2}}} \\ &= 4x^{-\frac{1}{2}} \end{aligned}$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use the rule $\frac{1}{a^m} = a^{-m}$
---	---

Practice questions

1 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$

b $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$

d $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$

f $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\frac{(2x^2)^3}{4x^0}$

h $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

2 Write the following as a single power of x .

a $\frac{1}{x}$

b $\frac{1}{x^7}$

c $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$

e $\frac{1}{\sqrt[3]{x}}$

f $\frac{1}{\sqrt[3]{x^2}}$

3 Write the following without negative or fractional powers.

a x^{-3}

b x^0

c $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$

e $x^{-\frac{1}{2}}$

f $x^{\frac{3}{4}}$

Answers

1 a $\frac{3x^3}{2}$

c $3x$

e $y^{\frac{1}{2}}$

g $2x^6$

b $5x^2$

d $\frac{y}{2x^2}$

f c^{-3}

h x

2 a x^{-1}

d $x^{\frac{2}{5}}$

b x^{-7}

e $x^{-\frac{1}{3}}$

c $x^{\frac{1}{4}}$

f $x^{\frac{2}{3}}$

3 a $\frac{1}{x^3}$

d $\sqrt[5]{x^2}$

b 1

e $\frac{1}{\sqrt{x}}$

c $\sqrt[5]{x}$

f $\frac{1}{\sqrt[4]{x^3}}$