Pearson
Edexcel

Complete the square in the form $p(x+q)^2 + r$

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 1   Write $x^2 + 6x + 4$ in the form $p(x + q)^2 + r$, where p, q and r are integers to be found.

$x^2 + 6x + 4$ $(x + 3)^2 - 9 + 4$ $(x + 3)^2 - 5$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x + \dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2 + c = 0$ **2** Simplify.

Example 2   Write $2x^2 - 7x + 4$ in the form $p(x + q)^2 + r$, where p, q and r are integers to be found.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \dfrac{7}{2}x\right) + 4$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \dfrac{b}{a}x\right) + c$
$2\left[\left(x - \dfrac{7}{4}\right)^2 - \left(\dfrac{7}{4}\right)^2\right] + 4$	**2** Now complete the square by writing $x^2 - \dfrac{7}{2}x$ in the form $\left(x + \dfrac{b}{2a}\right)^2 - \left(\dfrac{b}{2a}\right)^2$
$2\left(x - \dfrac{7}{4}\right)^2 - \dfrac{49}{8} + 4$	**3** Expand the square brackets.
$2\left(x - \dfrac{7}{4}\right)^2 - \dfrac{17}{8}$	**4** Simplify.

Practice questions

1   Write the following in the form $p(x + q)^2 + r$, where p, q and r are integers to be found

   a $9x^2 + 18x - 2$ **b** $5x^2 - 15x + 4$

   c $4x^2 + 8x + 2$ **d** $4x^2 - 20x + 15$

   e $2x^2 + 6x - 1$ **f** $10x^2 + 2x + 3$

Answers

1   **a** $9(x + 1)^2 - 11$ **b** $5\left(x - \dfrac{3}{2}\right)^2 - \dfrac{29}{4}$

       c $4(x + 1)^2 - 2$ **d** $4\left(x - \dfrac{5}{2}\right)^2 - 10$

       e $2\left(x - \dfrac{3}{2}\right)^2 - \dfrac{11}{2}$ **f** $10\left(x - \dfrac{1}{10}\right)^2 + \dfrac{29}{10}$