

Quadratic formula to solve quadratic equations

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a , b and c .

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$ $x = \frac{-6 \pm \sqrt{20}}{2}$ $x = \frac{-6 \pm 2\sqrt{5}}{2}$ $x = -3 \pm \sqrt{5}$ So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	<ol style="list-style-type: none">Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.Substitute $a = 1, b = 6, c = 4$ into the formula.Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$Simplify by dividing numerator and denominator by 2.Write down both the solutions.
----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$ $x = \frac{7 \pm \sqrt{73}}{6}$ <p>So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$</p>	<ol style="list-style-type: none"> 1 Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it. 2 Substitute $a = 3$, $b = -7$, $c = -2$ into the formula. 3 Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2. 4 Write down both the solutions.
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Practice questions

1 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$

b $2x^2 - 4x - 7 = 0$

2 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a , b and c are integers.

3 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Hint

Get all terms onto one side of the equation.

4 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Answers

1 a $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ b $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

2 $x = \frac{7 + \sqrt{41}}{2}$ or $x = \frac{7 - \sqrt{41}}{2}$

3 $x = \frac{-3 + \sqrt{89}}{20}$ or $x = \frac{-3 - \sqrt{89}}{20}$

4 a $x = \frac{7 + \sqrt{17}}{8}$ or $x = \frac{7 - \sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$