

Simplifying fractions

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1
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Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
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Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2$ $= 3^2$ $= 9$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ 2 Use $\sqrt[3]{27} = 3$
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Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	<ol style="list-style-type: none"> 1 Use the rule $a^{-m} = \frac{1}{a^m}$ 2 Use $4^2 = 16$
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Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
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Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	<ol style="list-style-type: none"> 1 Use the rule $a^m \times a^n = a^{m+n}$ 2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$
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Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged
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Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use the rule $\frac{1}{a^m} = a^{-m}$
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Example 9 Simplify $\frac{x^5}{x^2}$

$\frac{x^5}{x^2} = x^3$	use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
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Example 10 Simplify $6x^6 \times 3x^4$

$6x^6 \times 3x^4 = 18x^{10}$	$6 \times 3 = 18$ and then use the rule $a^m \times a^n = a^{m+n}$ to give $x^6 \times x^4 = x^{6+4} = x^{10}$
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Example 11 Simplify $(x^4)^2 \times 3x^5$

$(x^4)^2 \times 3x^5 = 3x^{13}$	$3 \times 1 = 3$ and then use the rule $(a^m)^n = a^{mn}$ following by to give $a^m \times a^n = a^{m+n}$ $(x^4)^2 \times x^5 = x^{4 \times 2} \times x^5$ $= x^8 \times x^5$ $= x^{8+5}$ $= x^{13}$
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Practice questions

1 Write as sums of powers of x .

a $\frac{x^5 + 1}{x^2}$

b $x^2 \left(x - \frac{1}{x} \right)$

c $x^{-4} \left(x^2 + \frac{1}{x^3} \right)$

d $\frac{6x^5 + 3x^4}{3x^2}$

e $\frac{5x^5 + 20x^4}{10x^2}$

f $\frac{7x^5 - 5x^4}{2x^6}$

Answers

1 a $x^3 + x^{-2}$

b $x^3 - x$

c $x^{-2} + x^{-7}$

d $2x^3 + x^2$

e $0.5x^3 + 2x^2$

f $3.5x^{-1} - 2.5x^{-2}$