

Expanding brackets and surds

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd \sqrt{b}
- To rationalise $\frac{a}{b + \sqrt{c}}$ you multiply the numerator and denominator by $b - \sqrt{c}$

Example 1 Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$ \begin{aligned} &(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) \\ &= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4} \\ &= 7 - 2 \\ &= 5 \end{aligned} $	<ol style="list-style-type: none"> Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$ Collect like terms: $\begin{aligned} &-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} \\ &= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0 \end{aligned}$
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Practice questions

1 Expand and simplify.

a $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

b $(3 + \sqrt{3})(5 - \sqrt{12})$

c $(4 - \sqrt{5})(\sqrt{45} + 2)$

d $(5 + \sqrt{2})(6 - \sqrt{8})$

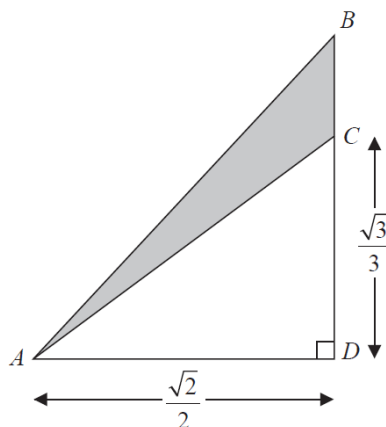
2 Expand and simplify $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

3 Work out the value of $(\sqrt{2} + \sqrt{8})^2$

4 Expand $(1 + \sqrt{2})(3 - \sqrt{2})$

Give your answer in the form $a + b\sqrt{2}$ where a and b are integers.

5 ABD is a right angled triangle.



All measurements are given in centimetres.

C is the point on BD such that $CD = \frac{\sqrt{3}}{3}$

$$AD = BD = \frac{\sqrt{2}}{2}$$

Work out the exact area, in cm^2 , of the shaded region.

- 6 The diagram shows a triangle DEF inside a rectangle $ABCD$.

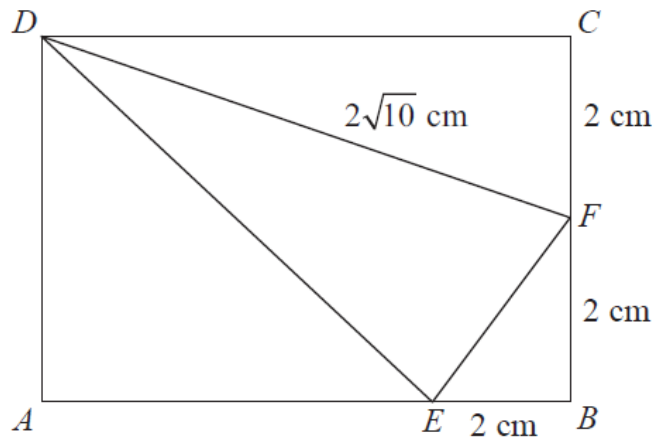


Diagram **NOT**
accurately drawn

Show that the area of triangle DEF is 8 cm^2 .
You must show all your working.

7 The diagram shows a triangle DEF inside a rectangle $ABCD$.

$$a = \sqrt{8} + 2$$

$$b = \sqrt{8} - 2$$

$$T = a^2 - b^2$$

Work out the value of T .

Give your answer in the form $c\sqrt{2}$ where c is an integer.

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Answers

1 a -1
c $10\sqrt{5}-7$

b $9-\sqrt{3}$
d $26-4\sqrt{2}$

2 $x-y$

3 18

4 $1+2\sqrt{2}$

5 $\frac{1}{4}-\frac{\sqrt{6}}{12}$

Method:

$$\frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \text{ or } \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3}$$

$$\frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3}$$

$$\frac{1}{4} - \frac{\sqrt{6}}{12} \text{ oe}$$

OR

$$(\text{BC} \Rightarrow) \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3}$$

$$\frac{1}{2} \times \left\{ \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3} \right\} \times \frac{\sqrt{2}}{2}$$

$$\frac{1}{4} - \frac{\sqrt{6}}{12} \text{ oe}$$

6 8

Method:

$$(2\sqrt{10})^2 - 2^2 (= 36)$$

$$(\text{CD} \Rightarrow) 6$$

$$'6' \times 4 - \frac{1}{2} \times '6' \times 2 - \frac{1}{2} \times 2 \times 2 - \frac{1}{2} \times ('6' - 2) \times 4$$

7 $16\sqrt{2}$