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Solving quadratic equations

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Example 1   Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$ Therefore $x = 0$ or $x = 3$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$. **2** Factorise the quadratic equation. $5x$ is a common factor. **3** When two values multiply to make zero, at least one of the values must be zero. **4** Solve these two equations.

Example 2   Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$ Therefore $x = -4$ or $x = -3$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) **2** Rewrite the b term ($7x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x + 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Example 3   Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\dfrac{4}{3}$ or $x = \dfrac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4   Solve $2x^2 - 5x - 12 = 0$

$b = -5, ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\dfrac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. $(-8$ and $3)$ **2** Rewrite the b term $(-5x)$ using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice questions

1   Solve

    a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

    c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

    e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

    g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

    i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

    k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2   Solve

    a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

    c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

    e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

    g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

> **Hint**
>
> Get all terms onto one side of the equation.

Answers

1 **a** $x = 0$ or $x = -\dfrac{2}{3}$ **b** $x = 0$ or $x = \dfrac{3}{4}$

    c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

    e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

    g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

    i $x = -7$ or $x = 4$ **j** $x = 3$

    k $x = -\dfrac{1}{2}$ or $x = 4$ **l** $x = -\dfrac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

    c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

    e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

    g $x = -3$ or $x = 2\dfrac{1}{2}$ **h** $x = -\dfrac{1}{3}$ or $x = 2$